MOTION OF A DROP IN A FLOW WITH VARYING VELOCITY

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An analytical solution is given for the problem concerning the motion of a drop in a gas flow, the velocity of which decreases linearly with distance.

A knowledge of the mechanisms of motion of a droplet and of particles in a flow of gas is of great importance for a large number of fields of technology concerned with dispersed systems. In particular, we may refer here to processes taking place in the combustion chambers of aircraft engines, in internal combustion engines, in chemical equipment, in technological facilities, etc. In the majority of cases, the dispersed phase is entrained by the continuum, the velocity of which is decreasing along the zone of contact. This nature of motion of the phases occurs, for example, in air-breathing jet engines [1, 2], in atomizing-type apparatuses [3], in various contact devices with unidirectional flow motion [4-8], in Venturi-type apparatuses [9], etc.

A knowledge of the velocity of the dispersed phases allows the stable diameter to be determined and the mechanism of the exchange process to be established; finally, it also allows recommendations to be made on the choice of structural parameters.

Because of this, the solution of the problem concerning the motion of a single drop in a stream of gas, the velocity of which decreases according to a linear law, is of undoubted interest. It should be noted that almost any nonlinear profile can be replaced by a piecewise-linear profile, i.e., this problem is of general importance.

We choose the following dependence of the gas velocity (W) on the distance (S) [10]:

$$W = W_0 - (W_0 - W_L)\,\frac{S}{L}. \tag{1}$$

The equation of motion of a spherical drop in a one-dimensional flow has the form

$$m_d\,\frac{dV_d}{d\tau} = \frac{\psi f_d \rho_g V^2}{2} \pm m_d g\left(1 - \frac{\rho_g}{\rho_d}\right). \tag{2}$$

The relative velocity of the drop is

$$V = W - V_d. \tag{3}$$

Let us consider three hydrodynamic regimes of flow of the drop.

Laminal Regime

In the case of slow movement of the drop (Re ≤ 1), the drag coefficient, as shown in [11], can be represented in the form

$$\psi = 24/\mathrm{Re}, \tag{4}$$

where $\mathrm{Re} = VD/\nu_g$; ν_g is the kinematic viscosity of the gas, m^2/sec. Transformation of Eq. (2), taking into account relations (1), (3) and (4), leads to an equation relative to S:

$$S'' + BS' + CS = F. \tag{5}$$

Here

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$$B = 18\,\mu_g/\rho_d D^2;\; C = 18\,\mu_g (W_0 - W_L)/\rho_d D^2 L;$$
$$F = 18\,\mu_g W_0/\rho_d D^2 \pm g\,(1 - \rho_g/\rho_d).$$

The general solution of Eq. (5) with the condition

$$S|_{\tau=0} = 0,\; S'|_{\tau=0} = V_{d0},$$

depending on the sign of the discriminant λ, has the form [12]

$$\lambda^2 = B^2 - 4C > 0,$$

$$S = C_1 \exp \frac{-B+\lambda}{2}\tau + C_2 \exp \frac{-B-\lambda}{2}\tau + \frac{F}{C}, \tag{6a}$$

where

$$C_1 = \frac{V_{d0}}{\lambda} - \frac{2F}{\lambda(B-\lambda)};\; C_2 = \frac{2F}{\lambda(B+\lambda)} - \frac{V_{d0}}{\lambda}.$$

$$\lambda^2 = 4C - B^2 > 0;$$

$$S = \exp\left(-\frac{1}{2}B\tau\right)\left(C_1 \cos\frac{1}{2}\lambda\tau + C_2 \sin\frac{1}{2}\lambda\tau\right) + \frac{F}{C}. \tag{6b}$$

Here

$$C_1 = -F/C;\; C_2 = 2V_{d0}/\lambda - FB/C\lambda;$$

$$4C = B^2;$$

$$S = \exp\left(-\frac{1}{2}B\tau\right)(C_1 + C_2\tau) + \frac{4F}{B^2}, \tag{6c}$$

where $C_1 = -4F/B^2$ and $C_2 = V_{d0} - 2F/B$.

Differentiating Eqs. (6a)-(6c), we determine the velocity of the drop: when $\lambda^2 = B^2 - 4C > 0$,

$$V_d = \frac{-B+\lambda}{2} C_1 \exp\frac{-B+\lambda}{2}\tau - \frac{B+\lambda}{2} C_2 \exp\frac{-B-\lambda}{2}\tau; \tag{7a}$$

when $\lambda^2 = 4C - B^2 > 0$,

$$V_d = \exp\left(-\frac{1}{2}B\tau\right)\left[V_{d0}\cos\frac{1}{2}\lambda\tau + \frac{1}{\lambda}(2F - V_{d0}B)\sin\frac{1}{2}\lambda\tau\right]; \tag{7b}$$

and when $4C = B^2$,

$$V_d = \exp\left(-\frac{1}{2}B\tau\right)\left[V_{d0} + \frac{1}{2}(2F - V_{d0}B)\,\tau\right]. \tag{7c}$$

The relative velocity of the drop at any instant of time, it can easily be seen, is determined by means of expressions (1), (3), (6), and (7).

Transition Regime

The most correct relation for determining the drag coefficient of a drop in a given regime is the equation obtained in the work of A. T. Litvinov, which takes account of the inertial terms [13, 14]:

$$\psi = a + b/\mathrm{Re}. \tag{8}$$

The numerical values of the coefficients a and b are given in [13, 14].

Substituting relations (1), (3), and (8) into Eq. (2) and neglecting the force of gravity together with the buoyancy, in consequence of their smallness in comparison with the strong aerodynamic action of the flow for $\mathrm{Re} > 1$ [2, 15], we obtain

$$S'' - AS'^2 + (C - ES)\,S' - FS^2 + QS - H = 0, \tag{9}$$

where

$$A = 3a\rho_g/4\rho_d D;\ B = 3b\mu_g/4\rho_d D^2;\ C = 2Ac + B;\ c = W_0;$$
$$E = 2Ah;\ h = (W_0 - W_L)/L;\ F = Ah^2;\ Q = 2Ach + Bh;$$
$$H = Ac^2 + cB.$$

The substitution of $p = S'$ reduces Eq. (9) to an Abel equation of second order [12]:

$$pp' = Ap^2 + (ES - C)\,p + FS^2 - QS + H,$$

which is insolvable in quadratures. Therefore, we shall use another approach to the solution of this problem, in which we replace Eq. (1) by the relation

$$W = c\exp(-h\tau), \tag{10}$$

obtained as a result of integrating Eq. (1) with the condition $S|_{\tau=0} = 0$. Then after substituting Eq. (10) in Eq. (3) and differentiating, we shall have

$$\frac{dV_d}{d\tau} = -hc\exp(-h\tau) - \frac{dV}{d\tau}. \tag{11}$$

The use of relations (8) and (11) converts Eq. (2), with the condition that $m_d g\,[1-(\rho_g/\rho_d)] = 0$, into Riccati's equation

$$\frac{dV}{d\tau} + AV^2 + BV = -hc\exp(-h\tau). \tag{12}$$

The corresponding boundary conditions have the form

$$V|_{\tau=0} = V_0.$$

In expressions (10)-(12), the coefficients A, B, c, and h are the same as in Eq. (9).

We then find the general solution of Eq. (12).

Using the well-known substitution [12]

$$V = -\frac{ht}{A}\cdot\frac{u'}{u},\quad t = \exp(-h\tau),$$

we obtain the equation

$$tu'' + (1 - N)\,u' + Eu = 0, \tag{13}$$

where $N = B/h$ and $E = cA/h$. Assuming further that

$$u = X^N Z(X),\quad X = 2\sqrt{Et},$$

we reduce Eq. (13) to a Bessel equation

$$X^2Z'' + XZ' + (X^2 - N^2)\,Z = 0,$$

the solution of which is well known [11]:

$$Z = C_1 J_N(X) + C_2 Y_N(X).$$

Here $J_N(X)$ and $Y_N(X)$ are N-th order Bessel functions, respectively, of the first and second kinds.

Reverting to the variables V and t, we obtain

$$V = -\frac{h\sqrt{Et}}{A}\cdot\frac{C_1 J_{N-1}(2\sqrt{Et}) + C_2 Y_{N-1}(2\sqrt{Et})}{C_1 J_N(2\sqrt{Et}) + C_2 Y_N(2\sqrt{Et})},$$

where $J_{N-1}(2\sqrt{Et})$ and $Y_{N-1}(2\sqrt{Et})$ are $(N-1)$-th order Bessel functions, respectively, of the first and second kinds.

The arbitrary constants C_1 and C_2 are determined by means of the following system of equations:

$$(2\sqrt{Et_0})^N\,[C_1 J_N(2\sqrt{Et_0}) + C_2 Y_N(2\sqrt{Et_0})] = V_0,$$
$$2E(2\sqrt{Et_0})^{N-1}\,[C_1 J_{N-1}(2\sqrt{Et_0}) + C_2 Y_{N-1}(2\sqrt{Et_0})] = -\frac{AV_0^2}{ht_0}.$$

Turbulent Regime

The drag coefficient of the drop in this case usually is assumed to be constant [16, 17]: ψ = const. Transformation of Eq. (2), taking Eq. (11) into consideration with the condition that $m_d g\,[1-(\rho_g/\rho_d)] = 0$, leads to Riccati's equation

$$\frac{dV}{d\tau} + AV^2 = -hc\exp(-h\tau), \tag{14}$$

where $A = 3\psi\rho_g/4\rho_d D$. The corresponding boundary conditions have the form

$$V|_{\tau=0} = V_0.$$

By means of the substitution

$$V = -\frac{ht}{A} \cdot \frac{u'}{u}, \quad t = \exp(-h\tau)$$

we reduce Eq. (14) to the form

$$tu'' + u' + Eu = 0,$$

where $E = Ac/h$.

The substitution of $X = 2\sqrt{Et}$ makes it possible to convert to the Bessel equation

$$X^2u'' + Xu' + X^2u = 0,$$

the solution of which is expressed in terms of a Bessel function of the first and second kinds and of zeroth order:

$$u = C_1J_0(X) + C_2Y_0(X).$$

Reverting to the variables u and t, we obtain

$$V = \sqrt{\frac{hct}{A}} \cdot \frac{C_1J_1(2\sqrt{Et}) + C_2Y_1(2\sqrt{Et})}{C_1J_0(2\sqrt{Et}) + C_2Y_0(2\sqrt{Et})}.$$

Here $J_1(2\sqrt{Et})$ and $Y_1(2\sqrt{Et})$ are first order Bessel functions, respectively, of the first and second kinds.

The arbitrary constants C_1 and C_2 are found from the system of equations

$$C_1J_0(2\sqrt{Et_0}) + C_2Y_0(2\sqrt{Et_0}) = V_0,$$
$$\sqrt{\frac{E}{t_0}}\,[C_1J_1(2\sqrt{Et_0}) + C_2Y_1(2\sqrt{Et_0})] = \frac{AV_0^2}{ht_0}.$$

NOTATION

W, running gas velocity, m/sec; W_0, W_L, gas velocities at the place of introduction of the droplet and at a distance L from it, m/sec; V_d, V, absolute and relative velocities of the drop, m/sec; V_{d0}, V_0, initial values of the absolute and relative velocities of the drop, m/sec; L, distance at which the velocity W_L is specified, m; S, path traversed by the drop, m; m_d, f_d, mass (kg) and the area of the center section (m^2) of the drop, respectively; g, acceleration due to gravity, m/sec^2; D, diameter of the drop, m; $Re = VD/\nu_g$, Reynold's criterion; J_N, J_{N-1}, J_1, J_0, Bessel functions of the first kind of order N, N − 1, 1, and 0, respectively; Y_N, Y_{N-1}, Y_1, Y_0, Bessel functions of the second kind of order N, N − 1, 1, and 0, respectively; $t = \exp(-h\tau)$, $X = 2\sqrt{Et}$, new independent variables; $p = S'$, u(t), Z(X), new dependent variables; t_0, initial value of an independent variable; A, B, C, E, F, H, M, N, Q, a, b, c, h, constant coefficients; C_1, C_2, arbitrary constants; λ, discriminant of Eq. (5); τ, time, sec; ρ_g, ρ_d, densities of the gas and of the drop, kg/m^3; ψ, hydrodynamic drag coefficient of the drop; ν_g, μ_g, kinematic (m^2/sec) and dynamic ($N \cdot sec/m^2$) viscosities of the gas.

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TWO-PHASE FLOWS WITH FRICTION

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Results are presented of a study of the equations of one-dimensional steady two-phase flows, taking account of friction with the channel walls.

§1. One-dimensional steady flows of a wet vapor in thermodynamic equilibrium are studied. The thermal conductivity of the vapor, the volume of the liquid phase, and the difference between the phase velocities are not taken into account.

It is assumed that friction is the only uncompensated external action on the flow. These flows belong to the class of flows with one internal degree of freedom [1] — the phase transition — and one external action — friction.

The effect of friction appears to one degree or another in all motions of two-phase media in channels. The pressure drop in a channel due to the performance of work against friction is an important engineering characteristic. Many empirical relations are known for calculating the pressure loss due to friction [2]. However, each of these has a limited range of application and does not reflect the dynamics of the flow of a wet vapor. It is of interest to study the appropriate differential equations to determine the general qualitative character of flows of a wet vapor acted upon by frictional forces following any resistance laws for all possible values of the parameters of a two-phase medium compatible with the conditions of the problem posed.

The results of the analysis can be applied to the little studied but practically important theoretical problem of the efflux of a self-evaporating liquid. This flow is, on the whole, nonequilibrium, but for a sufficiently long channel it has a quasiequilibrium boundary region of wet vapor [3]. The cross-sectional area of the channel occupied by the wet vapor and also its mass flow rate vary from section to section as a result of the vaporization of the metastable liquid at the center of the channel. The temperature of the liquid remains practically constant [3], and it will be shown later that this leads to the compensation of the geometric action of the emerging stream on the flow of wet vapor in the boundary region. The equations describing this flow are the same as the equations of equilibrium two-phase flow with friction.

§2. The equations of continuity, motion, and energy corresponding to the equilibrium flow of a wet vapor in a channel of constant cross section have the form

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